CHAPTER 2 Introduction to polarisation and scattering in optics

Light in general can be described both as a particle flow of photons as used in quantum mechanics and as a classical wave phenomenon. If described as an electromagnetic wave, it may be characterised by its brightness, colour and state and degree of polarisation. The human eye is a very sensitive and complex light sensor which enables us to see different brightness and colours in the visible wavelength range but is, in general, polarisation blind.

The phenomenon of polarised light was observed in 1669 by Bartholinus and independently in 1690 by Huygens who both observed the double refraction of light by calcite, known nowadays as birefringence. Malus in 1808 used, for the first time, the term 'polarisation' in describing the production of polarised light by reflection which he derived from the term ‘polarity’ as used to describe the two sides of magnetic poles. Polarisation in the sense as used by Malus, who tried to support the corpuscular theory, is a misnomer because he explained the polarisation of light as the polarity of the corpuscles. The proper description of polarised light had to wait until Fresnel and Young (circa 1816) who showed that light waves are of transverse nature and orthogonally polarised light cannot interfere. James Maxwell (1831-1879) unified the existing theories of electricity and magnetism leading to the predicted existence of electromagnetic waves which travel with the known speed of light, and concluded that light itself consisted of such waves.

The structure of this chapter is as follows. Section 2.1 introduces the electric field description of a plane wave which is the simplest solution of Maxwell’s equations. The state of polarisation (SOP) of the electric field will be visualised with the help of the polarisation ellipse. Section 2.2 discusses the necessary mathematical tools, such as Jones and Mueller calculus, for dealing with polarisation and its effects in birefringent mediums. In Section 2.3, the Poincaré sphere will be introduced which is the most frequently used tool for visualisation of the SOP and birefringence effects. In Section 2.4, the SOP for light travelling in forward - backward direction will be defined for Jones and Stokes vector in a reciprocal media. The definition will be consistent with the electric field definition for forward and backward directions and obeys the reciprocity theorem of reciprocal systems. Section 2.5 describes Fresnel reflection and Rayleigh scattering in optical fibres. The expected scattered intensity from silica is calculated. The aspect of the degree of polarisation of the received scattered SOP will be discussed.

2.1 Field representation of polarised light

Following Maxwell's equation for light propagation in a homogenous isotropic dielectric media, the simplest solution is a planar wave propagating in the positive z direction which can be written, in general, as two linear independent transverse fluctuations in the x and y direction [26]

$$\vec{\mathbf{E}}(z,t) = \begin{bmatrix} e_x \\ e_y \end{bmatrix} e^{j(\omega t - kz)} = \begin{bmatrix} A_x e^{j\phi_x} \\ A_y e^{j\phi_y} \end{bmatrix} e^{j(\omega t - kz)} \tag{2.1}$$

where A_x and A_y are the amplitudes, ϕ_x and ϕ_y the initial phases, $\omega = 2\pi f$ the circular frequency at the optical frequency f of the monochromatic wave and k is the wave number in (rad/m) which can be expressed in different ways as $k = \omega \bullet (\varepsilon\mu_0)^{1/2} = k_0 \bullet n = 2\pi n/\lambda_0$ with μ_0 the permeability, ε the permittivity, k_0 the free space wavenumber and n the refractive indices at the free space wavelength λ_0. For reverse propagation direction, $z = -z$ in Equation (2.1).

Assuming the electric field given in Equation (2.1) could be measured with an x-y oscilloscope [27] at $z = 0$, in general, a two dimensional ellipse as shown in Figure 2.1(a) would be observed. The polarisation ellipse is used to visualise the SOP in a two dimensional plane. The shape and orientation of the polarisation ellipse is defined by its azimuth θ and ellipticity ψ which can be calculated from the electric field in Equation (2.1) [28]

$$\begin{aligned} 2\theta &= \arctan\left(\frac{2 A_x A_y \cos(\phi)}{A_x^2 - A_y^2} \right) \\ 2\psi &= \arcsin\left(\frac{2 A_x A_y \sin(\phi)}{A_x^2 + A_y^2} \right) \end{aligned} \tag{2.2}$$

where $\phi = \phi_x - \phi_y$. The SOP in Figure 2.1(a) is left elliptical polarisation (LEP) characterised by a positive ellipticity. If we look towards the source in our right hand set co-ordinate system, the electric field rotates with time in a counter clockwise direction which we define in this thesis as LEP light. For $\psi = 0$, the ellipse collapses to a line and the SOP is linear (if $A_x = 0$ or $A_y = 0$ or $\phi_x = \phi_y$). The SOP is left and right circular polarised if $A_x = A_y$ and $\phi = \pm\pi/2$. The same electric field vector as used for the polarisation ellipse in Figure 2.1(a) traces along the *z* axis a left-handed screw viewed at an instant in time, Figure 2.1(b).

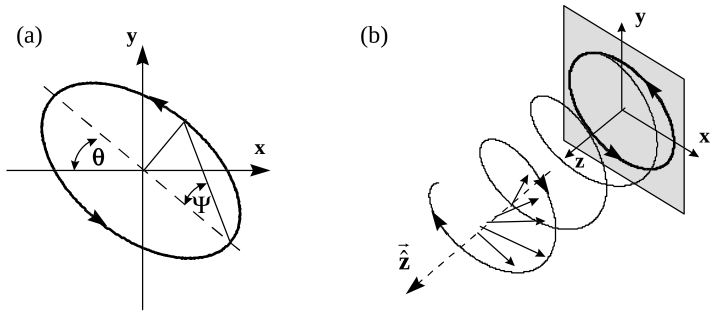

Figure 2.1 Representation of left elliptical polarised light (a) Polarisation ellipse looking towards the source (b) path of the electric field vector at a single instance of time.

2.2 Mathematical description of polarisation

Jones and Mueller calculus: The two calculus most often employed in classical optics are the Jones calculus and Stokes - Mueller calculus. In 1940-41, Jones [29] developed a matrix method of 2×2 complex matrices for mathematically treating polarisation effects in optical elements. He described linear optical systems which includes different types of birefringence and polarisation dependent loss, the reversibility of optical systems, etc. Jones calculus is in general used for fully polarised light. Mueller in 1943 [30], [31] described the same optical systems as Jones by using the Stokes calculus with a real 4×4 matrix description which can describe totally or partially polarised light. Jones - Stokes vectors and the corresponding matrices are generally independent of the direction of propagation in three dimensions.

Nowadays, the two main approaches employed in optics for describing the interaction of polarised light with optical devices are the Jones and Mueller calculus. Both are well explained in books like [28], [32].

2.2.1 Jones calculus

The Jones calculus assumes a harmonic transverse electromagnetic wave as the plane wave given in Equation (2.1). Since the SOP of the electric field in Equation (2.1) is completely determined by its initial phase and amplitude, the Jones vector defined in the positive direction is

$$\vec{\mathbf{J}} = \begin{bmatrix} e_x^+ \\ e_y^+ \end{bmatrix} = \begin{bmatrix} A_x e^{j\phi_x} \\ A_y e^{j\phi_y} \end{bmatrix} \tag{2.3}$$

The Jones vector contains complete information about the amplitudes and the phase of the electric field components. If only the state of polarisation of the wave is of interest, the Jones vector can be simplified by normalising the Jones vector to unity amplitude

$$\vec{\hat{\mathbf{J}}}_N = \vec{\mathbf{J}} \Big/ \sqrt{\vec{\mathbf{J}}\vec{\mathbf{J}}^*} \tag{2.4}$$

thereby losing the absolute phase and amplitude information of the two orthogonal fields but gaining a simpler expression for the vector. In books like [28], large tables of Jones vectors for different SOPs can be found. Two complex vectors are orthogonal if the scalar product with one of the vectors a transpose conjugate is zero. The same is true for two SOPs which are referred to as being orthogonal when their Jones states are orthogonal

$$\vec{\mathbf{J}}_a^{*T} \vec{\mathbf{J}}_b = 0 \tag{2.5}$$

The Jones matrix for a linear optical element assuming no differential loss between the two eigenmodes can be written as a 2×2 matrix $\mathbf{M}_J$ whose elements are in general frequency dependent [33], [34]

$$\vec{\mathbf{J}}_{out} = \mathbf{M}_J(\omega)\vec{\mathbf{J}}_{in} \tag{2.6}$$

with $\mathbf{M}_J = e^{-\left(\frac{1}{2}\alpha + i\beta_A(\omega)\right)z}\mathbf{U}$ where α is the absolute power attenuation coefficient, $\beta_A(\omega)$ is the propagation constant containing the absolute phase information (chromatic dispersion) and $\mathbf{U}$ is the Jones polarisation matrix of the optical element. In the absence of polarisation dependent loss, $\mathbf{U}$ is a unitary matrix $\mathbf{U}^T\mathbf{U}^* = \mathbf{I}$ where $\mathbf{I}$ is the identity matrix. The unitary matrix is of the following type

$$\mathbf{U} = \begin{bmatrix} u_1 & u_2 \\ -u_2^* & u_1^* \end{bmatrix} \quad with \quad |u_1|^2 + |u_2|^2 = 1 \tag{2.7}$$

The Jones matrix in backward direction, for a reciprocal system, is the transpose of the matrix in the forward direction [33]

$$\mathbf{M}_{J,B} = \mathbf{M}_{J,F}^T \tag{2.8}$$

where the second subscript means forward and backward direction.

2.2.2 Stokes calculus

The Stokes form is of practical importance because the four Stokes parameters represent intensities which are real ‘observable’ quantities in optics and can be directly measured with the use of a polarimeter and displayed on the Poincaré sphere. In reality, there is no monochromatic wave as assumed in Equation (2.1) and a more realistic assumption is a *quasi monochromatic wave* which can be introduced as a fluctuation of the amplitude *A(t)* and initial phase $\phi(t)$, but with the fluctuation small on a time scale compared to the frequency of the light. The Stokes intensities are related to the electric field of the quasi monochromatic plane wave as the time averaged intensities of the electric field, and can be measured through different polarisers

$$\vec{\mathbf{S}} = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} \langle e_x e_x^* + e_y e_y^* \rangle \\ \langle e_x e_x^* - e_y e_y^* \rangle \\ \langle 2\,\mathrm{Re}\, e_x e_y^* \rangle \\ \langle 2\,\mathrm{Im}\, e_x e_Y^* \rangle \end{bmatrix} = \begin{bmatrix} \langle A_x^2 + A_y^2 \rangle \\ \langle A_x^2 - A_y^2 \rangle \\ \langle 2 A_x A_y \cos(\phi) \rangle \\ \langle 2 A_x A_y \sin(\phi) \rangle \end{bmatrix} = \begin{bmatrix} \langle I \rangle \\ \langle H - V \rangle \\ \langle P - Q \rangle \\ \langle L - R \rangle \end{bmatrix} \tag{2.9}$$

where the brackets $\langle\ \rangle$ denotes time averaging over at least one optical frequency period, *I* is the total intensity, H, V, P and Q are the light intensity measured through linear polarisers orientated at 0°, 90°, +45° and -45° respectively and L, R are the intensities through a left-handed and right-handed circular polariser respectively.

The averaging in Equation (2.9) is necessary because, in general, light is not completely polarised and may be characterised by an electric field vector which at any instance in time has a well-defined state of polarisation that fluctuates randomly on a time scale which is slow compared to the frequency of light. For completely polarised coherent waves, monochromatic light is assumed, as for the Jones vector, the amplitude and phase are time independent and the averaging in Equation (2.9) can be dropped. The degree of polarisation is a measure of how well behaved the SOP is, and is defined as the intensity ratio of the polarised part to the total intensity

$$DOP = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0} \tag{2.10}$$

In a real system, the averaged Stokes vector will often depend on the integration time *T* of the measurement system which integrates the individual Stokes parameters with time

$$\langle S_n \rangle = \frac{1}{T}\int_0^T S_n(t)dt \qquad n = 0, 1, 2, 3 \tag{2.11}$$

The shortest integration time will normally be given by the receiver response time. The DOP may be determined by adjusting an arbitrary polariser[1] for minimum P_{min} and maximum P_{max} power transmission

$$DOP = \frac{\langle P_{\max} \rangle - \langle P_{\min} \rangle}{\langle P_{\max} \rangle + \langle P_{\min} \rangle} \tag{2.12}$$

which will be a useful equation in Chapter 7 when measuring the DOP of a short optical pulse. In reality, a light wave contains a whole frequency spectrum, with width depending on the source which may be broadened by modulation. If the spectral width broadens, the electric field amplitudes and phases in Equation (2.1) will vary more erratically with time. Hence the DOP will decrease and the SOP will be more difficult to define precisely if both modes are excited. For unpolarised light, DOP = 0, the electric field amplitudes and phases in Equation (2.1) averaged over time will be $\langle A_x^2 \rangle = \langle A_y^2 \rangle$ and $\langle \cos(\phi) \rangle = \langle \sin(\phi) \rangle = 0$ so that the Stokes vector for depolarised light is

$$\vec{\mathbf{S}}_U = \begin{bmatrix} 2\langle A_x^2 \rangle & 0 & 0 & 0 \end{bmatrix}^T \tag{2.13}$$

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[1] An arbitrary polariser may be formed, for example, by a λ/4 plate - linear polariser combination.

For describing partly polarised light the Jones vector is not very useful although it is possible by introducing the coherency matrix [26] which is an extension of the Jones calculus. The Stokes vector, on the other hand, naturally describes partly polarised light as a combination of two beams, one being completely polarised $\vec{\mathbf{S}}_P$ and the other completely unpolarised $\vec{\mathbf{S}}_U$

$$\vec{\mathbf{S}} = \vec{\mathbf{S}}_P + \vec{\mathbf{S}}_U = \begin{bmatrix} S_{0,P} \\ S_{1,P} \\ S_{2,P} \\ S_{3,P} \end{bmatrix} + \begin{bmatrix} S_{0,U} \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{2.14}$$

The Stokes components S_1, S_2 and S_3 of the polarised part in Equation (2.14) adequately characterise the state of polarisation and are often normalised with respect to the total intensity

$$\vec{\mathbf{s}} = \begin{bmatrix} S_{1,P} & S_{2,P} & S_{3,P} \end{bmatrix}^T / S_{0,P} \tag{2.15}$$

where $\|\vec{\mathbf{s}}\| = 1$. Two co-propagating Stokes vectors are mutually orthogonal if

$$\vec{\mathbf{s}}_a = -\vec{\mathbf{s}}_b \tag{2.16}$$

2.2.3 Mueller calculus

The Mueller matrix describes linear optical elements in a similar way to the Jones matrix but using a 4×4 matrix with only real components. Mueller matrices for retarders, polarisers and optical components can be found in [28] or [32] with derivation. An important feature of the Mueller matrix $\boldsymbol{M}_M$ for optical elements, with no polarisation dependent loss, is that it is an orthogonal matrix. A real orthogonal matrix has many mathematically helpful properties, e.g. its determinant is equal to plus one (det($\mathbf{M}_M$) = 1) and its inverse is equal to its transpose ($M_M^T = M_M^{-1}$). In Chapter 5, these properties and some more of the orthogonal matrices will be essential in the derivation of the fibre polarisation simulation. The Mueller matrix in the backward direction for a reciprocal system is similar to Equation (2.8) for the Jones matrix, the transpose of the matrix in the forward direction

$$\mathbf{M}_{M,B} = \mathbf{M}^{T}_{M,F} = \mathbf{M}^{-1}_{M,F} \tag{2.17}$$

It should be emphasised that Equation (2.17) is only true for birefringent mediums with no polarisation dependent loss and also ignores attenuation.

2.2.4 A comparison between Jones and Mueller calculus

Both methods have their advantages and disadvantages and complement each other to some extent. Jones treatment of polarisation is closely related to the electromagnetic wave theory and describes only fully polarised light, whereas Stokes and Mueller calculus can also treat partially polarised light and describe depolarisation. The Stokes vector is related to observable real quantities (intensities) which can be directly measured by using a polarimeter.

Jones calculus employs a smaller matrix and is applicable to systems where the absolute phase has to be preserved. For coherent light, the Jones calculus is more suitable in determining the SOP, as is the case in coherent scattering problems where two mutually coherent light beams have to be added by using the Jones vectors of the two beams. Only if the coherent beams have orthogonal states of polarisation may the Stokes vector be used to combine the vectors. On the other hand, for the more practical case of the combination of incoherent light beams, the individual Stokes vectors can be just added, whereas the Jones calculus requires an extension to the density matrix which is of greater complexity. Further, for depolarising optical systems, the Mueller calculus is superior to the Jones calculus [35] because every Jones matrix can be expressed as a Mueller matrix but not always vice versa as in the case of a depolariser.

2.3 The Poincaré sphere

The Poincaré sphere was introduced in 1892 by Poincaré as a geometrical representation of the Stokes vectors (developed by Stokes in 1852). The Poincaré sphere will be our most important tool to visualise the SOP throughout the thesis, and also to describe polarisation effects in optical elements.

2.3.1 The Stokes vector representation on the Poincaré sphere

The normalised Stokes components in Equation (2.15) are the co-ordinates on a unit sphere as indicated in Figure 2.2(c) with s_1, s_2 and s_3. Any SOP of the normalised Stokes vector $\vec{\mathbf{s}}$ given in Equation (2.15) can be directly represented as a unique point on the surface of the Poincaré sphere. Depolarisation can be represented on the sphere by using its radial dimension. A general elliptical SOP characterised by its azimuth θ and ellipticity ψ as shown in Figure 2.2(a) or Figure 2.1(a) is represented on the Poincaré sphere by a point SOP_A with longitude and latitude 2θ and 2ψ respectively as shown in Figure 2.2(c). Hence, all linear polarised states for which the ellipticity vanishes (if A_x =0 or A_y = 0 or $\phi_x = \phi_y$) are represented by points on the equator of the sphere.

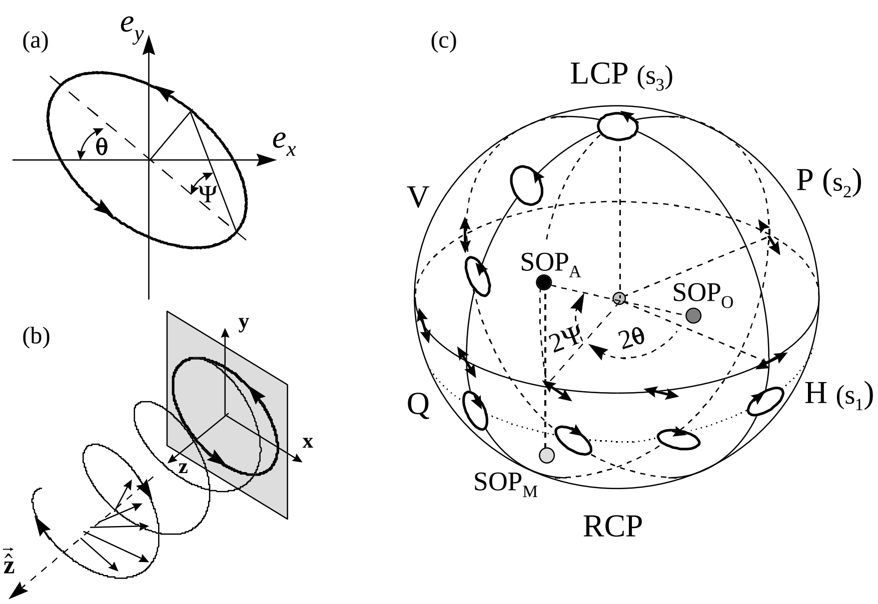

Figure 2.2 Polarisation ellipse and Poincaré sphere representation of SOP

The absolute direction of the linear states has to be defined with respect to a fixed laboratory frame. The poles of the sphere represent left and right circular polarisation (if $A_x = A_y$ and $\phi = \pm\pi/2$) where left circular polarisation has positive handedness and is always on top of the

Poincaré sphere. All the other possible SOPs on the sphere are elliptical. The relation between the normalised Stokes vector $\vec{\mathbf{s}}$ and the azimuth - ellipticity can be found on the Poincaré sphere

$$\vec{\mathbf{s}} = \begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} \cos(2\theta)\cos(2\psi) \\ \sin(2\theta)\cos(2\psi) \\ \sin(2\psi) \end{bmatrix} \tag{2.18}$$

Orthogonal SOPs as formulated in Equation (2.16) are quite useful because any SOP can be represented as a linear superposition of two waves with orthogonal SOPs. On the sphere, orthogonal SOPs are simply at diametrically opposite ends on the Poincaré sphere, as indicated in Figure 2.2(c) with SOP_A and SOP_O. On the polarisation ellipse, orthogonal SOPs have perpendicular major axes, the same ellipticity and opposite handedness. The Poincaré sphere is an important tool to visualise the SOP response of optical elements as in the case of a simple retarder which is a simple rotation of the Poincaré sphere around the eigenvector of the retarder.

2.3.2 The use of the Poincaré sphere to describe birefringent systems

There are two equivalent ways of describing polarisation effects in uniform birefringent mediums. One is by using the coupled mode theory [36]-[38] with the electric field vectors (Jones vector) and the other is by using the Mueller matrix description with Stokes vectors which can be directly related to a geometrical description on the Poincaré sphere [37]-[40]. The Mueller matrix for a birefringent element with no polarisation loss is, in general, an orthogonal matrix and can be represented on the Poincaré sphere as a rotation of the sphere around its eigenvectors, which are also called the polarisation eigenmodes, where the magnitude of the rotation is the birefringence of the optical element (Section 4.5). The Stokes vector and the Mueller matrix description of birefringent optical elements have been used throughout the thesis mainly because of their direct relation to, and simple visualisation on, the Poincaré sphere.

For the sake of completeness, it should also be mentioned that the SOPs, as mapped on the Poincaré sphere, can be projected onto a two dimensional complex plane to visualise and study the polarisation behaviour of optical systems [41], in a similar manner to the Poincaré sphere. Furthermore it is possible to use Jones matrices to describe a physical system and still use the Poincaré sphere to visualise the results.

2.4 Reciprocity in optical elements

The Jones vector for a wave propagating in the positive *z*-direction is defined in Equation (2.3). For a counter-propagating wave, the space inverse has to be taken which is the complex conjugate of the electric field in Equation (2.3). In bi-directional systems, the forward and backward matrices and SOPs have to be marked differently. Normally for the Jones and Mueller calculus, forward wave propagation direction is assumed. Next the Jones - Stokes vector for backward direction will be defined so that the reciprocity of the system is not invalidated.

2.4.1 Reciprocity theorem

Reciprocity may be stated in quite a general form that if a signal experiences a certain time delay and attenuation in one direction, it will experience the same attenuation and time delay in the reverse direction if the medium is symmetrical (reciprocal) with respect to the direction of propagation. For electromagnetic waves, the reciprocity theorem is often referred to as the Helmholtz reciprocity theorem. In optical fibres, in terms of polarisation, where our interest lies, it is useful to define the reciprocity with respect to the fibre polarisation modes and the mode coupling in the fibre. In a reciprocal system, the mode coupling from *mode A* at the fibre input to *mode B* at the output is the same as the mode coupling from *mode B* to *mode A* in the reverse direction, which is clearly not satisfied in the case of Faraday rotation[2] in the system.

2.4.2 Definition of Stokes vector for forward and backward direction

In Chapters 4 to 7, polarisation effects for forward and forward - reflected - backward travelling waves are treated using the Mueller calculus for media without polarisation loss. For the *forward direction,* the Mueller matrix is a simple rotation matrix where the actual form of the rotation matrix in the case of linear and circular birefringence will be derived in Chapter 5. On *reflection,* the incident SOP remains unchanged $\left(\hat{e}_x^+ = \hat{e}_x^- \text{ and } \hat{e}_y^+ = \hat{e}_y^-\right)$ as in the case for Rayleigh scattering, assuming ideal identical dipole scatterers and for Fresnel reflection from a glass to air interface (Section 2.5). However, because there is a change in

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[2] Faraday effect, e.g. in a Faraday rotator such as used in optical isolators, causes a non-reciprocal rotation of the SOP. A wave travelling in the +*z* direction will undergo the opposite rotation to a wave travelling in the –*z* direction with respect to the direction of propagation. This is in contrast to 'normal' birefringent mediums where the wave cannot differentiate between +*z* and –*z* propagation.

direction, we have to use the space reversal for the reflected SOP as will be seen below. For the *backward direction,* Jones [33] has shown that in the absence of magneto-optical rotation (Faraday effect), a birefringent medium is reciprocal and the backward Jones matrix is the transpose of the forward matrix which also holds for the Mueller calculus. In Section 4.5, the resultant measured Mueller matrices in forward, backward direction and forward - reflection - backward direction for different optical components will be shown.

The confusion starts when defining the Stokes vector for counter-propagating waves because the Stokes vector which is related to the intensity does not contain information about the direction of propagation. For the following derivation of the Stokes vector in backward direction, we use the convention for co-propagating waves as used in Reference [42] which is consistent with the principle of reciprocity. The backward direction for the Jones vector in Equation (2.3) is defined by the space reversal of the electric field in Equation (2.1) $\left(\vec{\hat{z}} = -\overleftarrow{\hat{z}}\right)$. For the Jones vector, we can say, in general, that the co-propagating optical waves have identical SOPs if $\vec{\mathbf{J}} = a\vec{\mathbf{J}}$ and for the Stokes vector if $\vec{\mathbf{s}}_1 = \vec{\mathbf{s}}_2$, where a is a complex constant. The identical SOP of the Jones vector for counter-propagating waves (as on reflection) with regard to its propagation direction is the space reversal or complex conjugate

$$\vec{\mathbf{J}} = a\overleftarrow{\mathbf{J}}^* \tag{2.19}$$

Following from that, the identical Stokes vector for a counter-propagating wave in the same sense as the Jones vector has to be the space reverse and from Equation (2.9) and (2.19) the reversed Stokes vector is

$$\overleftarrow{\mathbf{S}} = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} \left\langle e_x^* e_x + e_y^* e_y \right\rangle \\ \left\langle e_x^* e_x - e_y^* e_y \right\rangle \\ \left\langle 2\,\mathrm{Re}\, e_x^* e_y \right\rangle \\ \left\langle 2\,\mathrm{Im}\, e_x^* e_y \right\rangle \end{bmatrix} = \begin{bmatrix} \left\langle e_x e_x^* + e_y e_y^* \right\rangle \\ \left\langle e_x e_x^* - e_y e_y^* \right\rangle \\ \left\langle 2\,\mathrm{Re}\, e_x e_y^* \right\rangle \\ \left\langle -2\,\mathrm{Im}\, e_x^* e_y \right\rangle \end{bmatrix} = \begin{bmatrix} \left\langle A_x^2 + A_y^2 \right\rangle \\ \left\langle A_x^2 - A_y^2 \right\rangle \\ \left\langle 2 A_x A_y \cos(\phi) \right\rangle \\ \left\langle -2 A_x A_y \sin(\phi) \right\rangle \end{bmatrix} \tag{2.20}$$

From Equation (2.20), it can be seen that physically the complex conjugate means the reverse of the handedness of the polarisation ellipse (SOP). The handedness in that sense is defined with respect to the direction of propagation. Considering now the Mueller matrix for ideal reflection we can write [43]

$$\mathbf{R}_M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \tag{2.21}$$

On the Poincaré sphere the change of the handedness is a simple reflection of the SOP on the equatorial plane of the sphere as indicated in Figure 2.2 by the reflection of the SOP_A to SOP_M.

Reciprocal and non reciprocal optical elements

Optical fibres, couplers, erbium and semiconductor optical amplifiers can be regarded as reciprocal optical elements, while Faraday rotators and Brillouin amplifiers, which only amplify signals opposing to the pump wave and any optical system which possesses dichroism as in the extreme case of a polariser, are non-reciprocal optical elements. Optical fibres may show some non-reciprocity if there is a fast polarisation change along the fibre, e.g. in a very long system, the birefringence may change before the reflected wave travels backwards. This case, which is called dynamical reciprocity and investigated in Reference [44], is certainly only interesting in long length systems, whereas our measurements are taken on short lengths of fibres where there is little change in the measured output SOP with time.

2.5 Fresnel reflection and Rayleigh scattering

In this section, the scattering of light in optical fibres will be introduced and the main focus will be on the intensity of the backscattered light in glass. Scattering itself is quite complicated, especially if many anisotropic scatterers with some unknown statistic have to be considered for the net scattering intensity.

2.5.1 Elastic scattering dipole description

In optical fibres, four types of scattering losses can be observed which are Mie scattering (Fresnel reflection), Rayleigh scattering, Brillouin scattering and Raman scattering. In single mode fibres, as treated in Chapter 3 and onwards, the only relevant scattering directions are the forward and backward directions along the fibre axis and for that reason we are only interested in those directions. *Mie scattering and Rayleigh scattering* (actually Mie scattering includes Rayleigh scattering as a special case) are, in general, elastic scattering where the wavelength and the backscattered SOP are not altered on reflection [45]. In a classical way, elastic scattering can be described by elementary oscillators (dipoles) where the electronic charge bound to the nucleus is forced into oscillation by the alternating electric field of the incident light and emits or 'scatters' light. *Brillouin - and Raman scattering* are inelastic scattering and both show a frequency shift in the scattered light. In fused silica fibres, Brillouin and Raman scattering is, in general, quite weak compared to Rayleigh scattering unless stimulated by high input powers. Stimulated Brillouin and Raman scattering are fundamental non-linear effects and can cause limitations in multi wavelength transmission systems through multichannel cross talk and non-linear power loss [46].

2.5.2 Mie scattering and Fresnel reflection

Mie scattering, named after Gustav Mie (1908), occurs when light interacts with particles whose size is >>λ. The scattering acts coherently with the incident light. Mie scattering is strongly affected by the size, shape, refractive index and absorption of the scatterer medium. The scattering centres within the particle are closely arranged within one wavelength (e.g. in a crystal) and act coherently so as to cancel all scattered light except in the forward and backward direction. The net scattered intensity for this coherent scattering is normally much larger than Rayleigh scattering and has to be avoided if the Rayleigh scattered intensity is to be measured. From Mie scattering, the usual laws of reflection and refraction, such as Fresnel reflection, can be obtained. A typical Fresnel reflection occurs at a fibre to air interface or on a reflection from an optical mirror. The fraction of power that is reflected P_R for light at normal incidence P_I on the interface between two isotropic dielectric media with different refractive indices n_1 and n_2 is given by the power reflection factor [26]

$$r^2 = \frac{P_R}{P_I} = \left(\frac{n_1 - n_2}{n_1 + n_2} \right)^2 \tag{2.22}$$

The maximum backscattered power for an air to glass transition or vice-versa with n_1 =1.5 and n_2 = 1 for silica and air respectively is 4% of the incident power (≈ –14 dB).

2.5.3 Rayleigh scattering

The blue sky at day time is a nice example of Rayleigh scattering, named after Lord Rayleigh. His theory of scattering, in 1871, showed that the intensity of the scattered light is inversely proportional to the fourth power of the wavelength ($I \propto \lambda^{-4}$), which explains the blue of the sky due to the higher scatter effect of the air molecules in the outer atmosphere at shorter wavelengths. Rayleigh scattering is independent on the shape of the scatterer and is treated within Mie scattering as a special case. In general, the scatterer size is small compared to the incident wavelength and the scatterers are assumed as 'very' weak allowing the Born approximation which states that the incident field is not altered by the presence of the scatterer.

Inhomogenous materials like, *fused silica in optical fibres*, are ideal for Rayleigh scattering because the scattering in different directions is not cancelled due to interference by adjacent dipoles as in homogenous materials. The Rayleigh scattering in glasses arises mainly from the microscopic variations in the material density (random molecular structure), structural defects, inhomogenities and randomly located dopants. All these effects act together as a local refractive index change which is impressed in the fibre during the fibre drawing process. Rayleigh scattering from the point of attenuation determines the lowest possible loss in the fibre.

Similar to the received intensity in coherent detection with signal and local oscillator we can write down a general equation for the Rayleigh scattered intensity considering N independent scatterers (oscillators) from a volume V as shown in Figure 2.3. Assuming the general case of a quasi-monochromatic wave incident in such a volume, the scattered intensity can be expressed by a one dimensional fibre model considering just one of the two linear polarised orthogonal modes without losing generality as [45]

$$\begin{aligned} I_x &\propto \left(\sum_{i=1}^{N} \left\langle e_{x,i}(t) \right\rangle \right)^2 \\ &= \frac{1}{2} \sum_{i=1}^{N} \left\langle \mathrm{Re}\left(e_{x,i}(t) e_{x,i}^{*}(t) \right) \right\rangle + \sum_{i \neq j} \left\langle \mathrm{Re}\left(e_{x,i}(t) e_{x,j}^{*}(t) \right) \right\rangle \\ &= \frac{1}{2} \sum_{i=1}^{N} \left\langle A_{x,i}(t) \right\rangle^2 + \sum_{i \neq j} \mathrm{Re}\left(\left\langle A_{x,i}(t) A_{x,j}(t) e^{i\left(\phi_{x,i}(t) - \phi_{x,j}(t)\right)} \right\rangle \right) \end{aligned} \tag{2.23}$$

where A and ϕ are the amplitudes and absolute phases of the scattered field as seen at some receiver ends. In Equation (2.23), the scattered intensities are split into a phase insensitive term and a phase sensitive term. The phase sensitive term which determines the fraction of power which can interfere depends strongly on the light coherence length within the considered volume and the relative phase difference (length difference) of the randomly distributed scatterers in the considered volume.

The classical case of Rayleigh scattering in fibre assumes incoherent scattering which we will discuss in the next subsection. For incoherent scattering with negligible interference from the coherent part in Equation (2.23), we can think of a few cases: (*i*) a large volume with many independent scatterers so the coherent intensity will cancel due to successive interference, (*ii*) a light source with short coherence length (broad linewidth) so that the phase $\phi(t)$ changes erratically on a small time scale short compared to the measurement time and (*iii*) many independently measured scattered intensities (e.g. many light pulses) from the volume so that the coherent component cancels over time. Coherent scattering is mainly a problem in OTDRs employing highly coherent light sources and a coherent detection scheme, whereas in conventional direct detection OTDRs the effect is small.

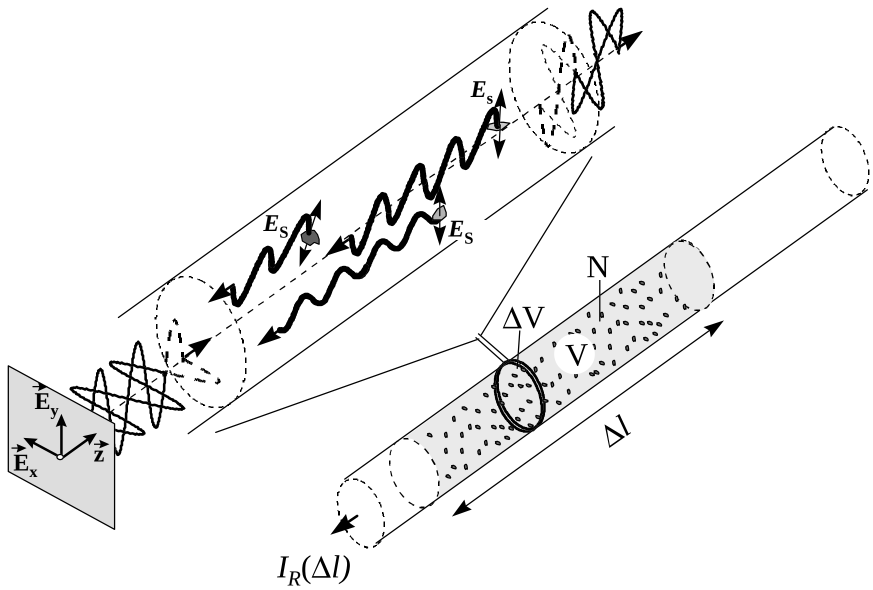

Figure 2.3 Rayleigh backscatter I_R in optical fibre from a volume *V*

2.5.4 Mean value of backscattered light

For incoherent scattering, there are many independent scatterers involved and statistical methods for the evaluation of the mean scattered intensity can be used [47], [48]. For frozen glass, the mean square of the density fluctuation -variance is directly related to the refractive index fluctuation and can be estimated from the thermodynamic density fluctuation in the melt at the solidification temperature (fictive temperature). The Rayleigh scattering loss α_S in (km^{-1}) due to density fluctuation (no dopant) as a function of the fictive temperature is given in Reference [49] and is defined as the ratio of the overall averaged scattered power to the average incident power

$$\alpha_S = \frac{8\pi^2}{3\lambda^4} n^8 p_{12}^2 K_B T_F \beta_T \tag{2.24}$$

where Δl is the scatter volume length, n is the refractive index, p_{12} = 0.27 is the photoelastic coefficient for silica, $K_B = 1.38 \bullet 10^{-23}$ Ws/K is the Boltzmann's constant, $T_F \approx 1600$ K is the solidifying temperature for pure silica, $\beta_T = 6.9 \bullet 10^{-11}$ s^2m/kg the isothermal compressibility [48]. The Rayleigh scattering loss at λ = 1.55 μm with n = 1.5 assuming no doping can be calculated from Equation (2.24) and is α_S = 0.04 /km. The backscatter loss is often given in dB/km $\alpha_{S,\,dB} = -10 log_{10}(1 - \alpha_S)$ which gives $\alpha_S \approx 0.18$ dB/km which is about the minimum Rayleigh scatter loss for the fictive temperature of pure silica. In optical fibres, the fibre core is often doped, for example, with GeO_2 (see Chapter 3) so that the light is guided in the core. The doping contributes to the scattering loss from the density fluctuation due to concentration fluctuation. For doped glass with for example GeO_2, T_F is decreasing but the total Rayleigh scattering coefficient which is a combination of density and concentration fluctuation increases linearly with the dopant concentration [50].

In Equation (2.24) the random medium is characterised by the thermodynamic density fluctuation. The same can be expressed by considering the statistical refractive index variation in a considered volume with refractive index variance $\langle \Delta n^2 \rangle$ and correlation length l_{cor} of Δn [51]

$$\alpha_S = b \frac{\langle \Delta n^2 \rangle l_{cor}^3}{\lambda^4} \tag{2.25}$$

where b is in practice just a constant depending on the approximation used for the random medium and its correlation function. In Equation (2.25), as in Equation (2.24), the scatter volume or length $\Delta l >> \lambda$ to ensure many independent scatterers are involved and the correlation length of Δn is assumed to be $l_{cor} << \lambda$.

2.5.5 Depolarisation of scattered light

When measuring polarisation effects, as for example, the polarisation matrix of a fibre in forward or forward-backward direction, the DOP should always be chosen as high as possible (close to 100%) in order to measure a well defined SOP, thus reducing the error in the SOP in the presence of noise (see Section 4.4). For this reason, it is important to understand the cause of depolarisation in optical fibres and how to avoid it. Next a list of possible depolarisation causes in the backscattered intensity is given which may be stronger or weaker depending on the used source and receiver condition

(i) Depolarisation in the forward direction is mainly caused by the polarisation dispersion, which is treated in Chapter 3, which starts to strongly affect the transmitted light if the delay between the transit time of the two orthogonal modes becomes greater than the coherence time of the source. If the light gets depolarised in the forward direction, the backward direction could, in general, double the depolarisation effect.

(ii) Coherent scattering leads to a fluctuation in the measured intensity. This may lead to an error in the measured SOP calculated from the measured intensities through different polarisers, if the fluctuation changes during the averaging process. The error in the measured SOP will then show up in the DOP.

(iii) A large pulse width, compared to the change of SOP along the fibre, can lead to depolarisation in the backscattered light due to an integration over all the backscattered SOPs within half the pulse volume (see Subsection 7.5.3).

(iv) In all the treatments above, ideal isotropic scatterers (spherical scatterers) have been assumed and this seems to be an acceptable approximation for optical fibres. It should be also mentioned that in remote sensing of individual particles, which also deals with Rayleigh scattering, the depolarisation effect of scattered light is used to determine the particles deviation from ideal sphericity [45], [52].

(v) Any error induced by the Stokes analyser unit for measuring the SOP (as for example, error in optical alignment, receiver noise and optical noise) leads to an uncertainty in the measured SOP as will be discussed in Section 4.4. This uncertainty reduces the measured DOP and increases if the measured light is only partly polarised.

The coherent fluctuation (ii) which decreases the DOP of the measured backscattered SOP can be reduced by measuring over many pulses (which also recovers the signal from the noise) but if the average period is too long the fluctuation, the input SOP or birefringence along the fibre may change and decrease the DOP again. Another more obvious way to reduce the coherent fluctuation is by the use of a broad band source or some modulation of the coherent light source to increase the linewidth, but if the birefringence is too high, depolarisation as mentioned in (i) may again occur.